MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 4 Mark Scheme

1) Find $\dfrac{dy}{dx}$ for each of the following. For (i) to (n) give the answer in terms of the parameter.

* d) $y = \cos(\ln x)$ [1 mark]

$$\frac{dy}{dx} = -\frac{\sin(\ln x)}{x}$$

* f) $y = \dfrac{1}{\cos^{-1} x}$ [1 mark]

$$\begin{aligned}
\frac{dy}{dx} &= -(\cos^{-1} x)^{-2} \frac{d}{dx}(\cos^{-1} x) \\
\text{Let } z &= \cos^{-1} x \\
x &= \cos z \\
\frac{dx}{dz} &= -\sin z \\
\frac{dz}{dx} &= -\frac{1}{\sin(\cos^{-1} x)} \\
\frac{dz}{dx} &= -\frac{1}{\sqrt{1-x^2}} \\
\text{Hence } \frac{dy}{dx} &= (\cos^{-1} x)^{-2} \frac{1}{\sqrt{1-x^2}} \\
&= \frac{1}{\sqrt{1-x^2}\,(\cos^{-1} x)^2}
\end{aligned}$$

* h) $y = e^{\ln x}$ [1 mark]

$$\begin{aligned}
y &= e^{\ln x} \\
&= x \\
\frac{dy}{dx} &= 1
\end{aligned}$$

* j) $x = \sin\theta$, $y = \cos\theta$ [1 mark]

$$\begin{aligned}
\frac{dx}{d\theta} &= \cos\theta \\
\frac{dy}{d\theta} &= -\sin\theta \\
\frac{dy}{dx} &= \frac{-\sin\theta}{\cos\theta} \\
&= -\tan\theta
\end{aligned}$$

* k) $x = \sin s$, $y = s$ [1 mark]

$$\begin{aligned}\frac{dx}{ds} &= \cos s \\ \frac{dy}{ds} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\cos s} \\ &= \sec s\end{aligned}$$

2) Find the following:

* e) $\displaystyle\int \frac{1}{\sqrt{x}}\ dx$ [1 mark]

$$\begin{aligned}\int \frac{1}{\sqrt{x}}\ dx &= \int x^{-\frac{1}{2}}\ dx \\ &= 2x^{\frac{1}{2}} + c\end{aligned}$$

* h) $\displaystyle\int_{\ln 1}^{\ln 11} e^x\ dx$ [1 mark]

$$\begin{aligned}\int_{\ln 1}^{\ln 11} e^x\ dx &= [e^x]_{\ln 1}^{\ln 11} \\ &= \left(e^{\ln 11}\right) - \left(e^{\ln 1}\right) \\ &= (11) - (1) \\ &= 10\end{aligned}$$

* j) $\displaystyle\int_0^{\frac{\pi}{2}} 10\sin x\ dx$ [1 mark]

$$\begin{aligned}\int_0^{\frac{\pi}{2}} 10\sin x\ dx &= [-10\cos x]_0^{\frac{\pi}{2}} \\ &= \left(-10\cos\tfrac{\pi}{2}\right) - (-10\cos 0) \\ &= (0) - (-10) \\ &= 10\end{aligned}$$

* k) $\displaystyle\int_1^2 \frac{20}{x^2}\ dx$ [1 mark]

$$\begin{aligned}\int_1^2 \frac{20}{x^2}\ dx &= \int_1^2 20x^{-2}\ dx \\ &= \left[-20x^{-1}\right]_1^2 \\ &= \left(-20\cdot 2^{-1}\right) - \left(-20\cdot 1^{-1}\right) \\ &= (-10) - (-20) \\ &= 10\end{aligned}$$

* l) $\int_{-2}^{3} 2x + 1 \; dx$ [1 mark]

$$\begin{aligned}\int_{-2}^{3} 2x + 1 \; dx &= \left[x^2 + x\right]_{-2}^{3} \\ &= \left(3^2 + 3\right) - \left((-2)^2 + -2\right) \\ &= (12) - (2) \\ &= 10\end{aligned}$$