MATH6103 Differential & Integral Calculus
MATH6500 Elementary Mathematics for Engineers

Problem Sheet 2 Mark Scheme

1) Without a calculator, find the value of $\tan\theta$ when:

* a) $\cot\theta = \frac{1}{2}$ [1 mark]

$$\begin{aligned}\tan\theta &= \frac{1}{\cot\theta}\\ &= 2\end{aligned}$$

* c) $\sec\theta = 4$ and $\tan\theta$ is positive [1 mark]

$$\begin{aligned}\tan^2\theta &= \sec^2\theta - 1\\ &= 15\\ \tan\theta &= \sqrt{15}\end{aligned}$$

* d) $\cos\theta = \frac{3}{5}$ and $\sin\theta$ is negative [1 mark]

$$\begin{aligned}\sin\theta &= \sqrt{1-\cos^2\theta}\\ &= \sqrt{1-\frac{9}{25}}\\ &= \sqrt{\frac{16}{25}}\\ &= -\frac{4}{5}\\ \tan\theta &= \frac{\sin\theta}{\cos\theta}\\ &= \frac{-4/5}{3/5}\\ &= -\frac{4}{3}\end{aligned}$$

2) Write each of the following in the form 2^r, where $r \in \mathbb{R}$:

* a) $\sqrt{2}$ [$\frac{1}{2}$ mark]

$$\sqrt{2} = 2^{\frac{1}{2}}$$

* c) $\sin\frac{\pi}{4}$ [$\frac{1}{2}$ mark]

$$\begin{aligned}\sin\frac{\pi}{4} &= \frac{1}{\sqrt{2}}\\ &= 2^{-\frac{1}{2}}\end{aligned}$$

* d) $3 \times 2^4 - 2^4$ [$\frac{1}{2}$ mark]

$$\begin{aligned} 3 \times 2^4 - 2^4 &= 2 \times 2^4 \\ &= 2^5 \end{aligned}$$

* f) 1 [$\frac{1}{2}$ mark]

$$1 = 2^0$$

* 3) Find all solutions to $(x^2 - 9x + 9)^{(x^2+x-6)} = 1$. [2 marks]

$$\begin{aligned} \text{Case 1: } x^2 - 9x + 9 &= 1 \\ x^2 - 9x + 8 &= 0 \\ (x-8)(x-1) &= 0 \\ x &= 1 \text{ or } 8 \\ \text{Case 2: } x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x &= -3 \text{ or } 2 \\ \text{Case 3: } x^2 - 9x + 9 &= -1 \text{ and } x^2 + x - 6 \text{ is even} \\ x^2 - 9x + 10 &= 0 \\ x &= \frac{-9 \pm \sqrt{41}}{2} \end{aligned}$$

At the values of x in case 3, $x^2 + x - 6$ is not a whole number, so case 3 has no solutions.
All solutions: $x = -3, 1, 2, 8$

[Atheeta: Give full marks if case 3 is ignored, as long as all four solutions found.]

4) Using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ for each of the following:

* c) $f(x) = g(x) + h(x)$ [2 marks]

$$\begin{aligned} f'(x) &= \lim_{k \to 0} \frac{f(x+k) - f(x)}{k} \\ &= \lim_{k \to 0} \frac{g(x+k) + h(x+k) - g(x) - h(x)}{k} \\ &= \lim_{k \to 0} \frac{g(x+k) - g(x)}{k} + \lim_{k \to 0} \frac{h(x+k) - h(x)}{k} \\ &= g'(x) + h'(x) \end{aligned}$$

* d) $f(x) = x^2$ [1 mark]

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \to 0} (2x + h) \\ &= 2x \end{aligned}$$