

MATH6103 Differential & Integral Calculus

Problem Sheet 1 Mark Scheme

1) Solve the following equations:

\* b)  $5x - 11 = 0$  [1 mark]

$$x = 11/5 \text{ ( or 2.2)}$$

\* e)  $x^2 - 3x = 80 - x$  [1 mark]

$$\begin{aligned}x^2 - 2x - 80 &= 0 \\(x + 8)(x - 10) &= 0 \\x &= -8 \text{ or } 10\end{aligned}$$

\* f)  $3x^2 + x - 2 = 0$  [1 mark]

$$\begin{aligned}x &= \frac{-1 \pm \sqrt{1 - 4 \times 3 \times (-2)}}{6} \\&= -1 \text{ or } \frac{2}{3}\end{aligned}$$

2) Solve the following equations:

\* b)  $x^2 + 2x + 5 = 0$  [1 mark]

$$\begin{aligned}x &= \frac{-2 \pm \sqrt{4 - 4 \times 5}}{2} \\&= -1 + 2i \text{ or } -1 - 2i\end{aligned}$$

\* c)  $5x^2 = 2x - 1$  [1 mark]

$$\begin{aligned}5x^2 - 2x + 1 &= 0 \\x &= \frac{2 \pm \sqrt{4 - 4 \times 5}}{10} \\&= \frac{1}{5} \pm \frac{2i}{5}\end{aligned}$$

\* d)  $x^3 + 4x^2 + x - 6 = 0$  [1 mark]

$$\begin{aligned}(x - 1)(x^2 + 5x + 6) &= 0 \\(x - 1)(x + 2)(x + 3) &= 0 \\x &= -3 \text{ or } -2 \text{ or } 1\end{aligned}$$

\* f)  $x^3 - 2x^2 - 4x = 0$  [2 marks]

$$x(x^2 - 2x - 4) = 0$$

$$x = 0 \text{ or } x^2 - 2x - 4 = 0$$

$$x^2 - 2x - 4 = 0$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = 1 \pm \sqrt{5}$$

$$\text{overall, } x = 0 \text{ or } 1 + \sqrt{5} \text{ or } 1 - \sqrt{5}$$

\* 3) Use completing the square on the equation  $x^2 + 2x + c = 0$  to show that  $x = -1 \pm \sqrt{1 - c}$ . [2 marks]

$$x^2 + 2x + c = 0$$

$$(x + 1)^2 - 1 + c = 0$$

$$(x + 1)^2 = 1 - c$$

$$x + 1 = \pm \sqrt{1 - c}$$

$$x = -1 \pm \sqrt{1 - c}$$