Problem Sheet 3 Mark Scheme

1) Differentiate the following functions:

* a) $a(x) = x^2 - \frac{1}{x^2}$ [1 mark]

$$a'(x) = 2x + 2x^{-3}$$

* d) $d(x) = 359x^{17}$ [1 mark]

$$d'(x) = 6103x^{16}$$

* f) $f(x) = \sin(x^2)$ [1 mark]

$$f'(x) = 2x\cos(x^2)$$

* g) $g(x) = xe^x$ [1 mark]

$$g'(x) = e^x + xe^x$$

2) Differentiate the following functions:

* a) $i(x) = \cos(4 + 3x^2)$ [1 mark]

$$i'(x) = -6x\sin(4 + 3x^2)$$

* d) $l(x) = \cos(\sin x)$ [1 mark]

$$l'(x) = -\sin(\sin x)\cos x$$

* g) $o(x) = \sqrt{\sin x + \cos x}$ [1 mark]

$$o'(x) = \tfrac{1}{2}(\sin x + \cos x)^{-\frac{1}{2}}(\cos x - \sin x)$$
$$= \frac{\cos x - \sin x}{2\sqrt{\sin x + \cos x}}$$

3) Find the x co-ordinates of the turning points of the following:

* b) $s(x) = x^3 - 108x$ [1 mark]

$$s'(x) = 3x^2 - 108$$
$$3x^2 - 108 = 0$$
$$x^2 = 36$$
$$x = \pm 6$$

* c) $r(x) = x^3 + 3x^2 + 2x - 8$ [1 mark]

$$r'(x) = 3x^2 + 6x + 2$$
$$3x^2 + 6x + 2 = 0$$
$$x = \frac{-6 \pm \sqrt{12}}{6}$$
$$x = \frac{-3 \pm \sqrt{3}}{3}$$

* d) $t(x) = \sin x + \cos x, \ -\frac{\pi}{2} < x < \frac{\pi}{2}$ [1 mark]

$$t'(x) = \cos x - \sin x$$
$$\cos x - \sin x = 0$$
$$\cos x = \sin x$$
$$1 = \tan x$$
$$x = \frac{\pi}{4}$$