

MATH6103 Differential & Integral Calculus Practice
Exam Paper

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All questions may be attempted but only marks obtained on the best **five** solutions will count.

The use of an electronic calculator is **not** permitted in this examination.

1. (a) Differentiate the following functions with respect to x .

(i) $f(x) = x \cos e^x$

(ii) $f(x) = (\ln x)^2$

(iii) $f(x) = \frac{1}{x^2} + 2x^3$

- (b) A curve is given by:

$$x = t^3 - 2\sqrt{t}$$

$$y = \tan t$$

Find $\frac{dy}{dx}$ in terms of t .

2. A function f is defined by

$$f(x) = x \sin(x).$$

(a) Find $f'(x)$.

(b) Find $f''(x)$.

(c) (i) Find a function $g(x)$ such that $g'(x) = f(x)$.

(ii) Write down a function $h(x)$ such that $h'(x) = f(x)$ and $g(x) \neq h(x)$.

(d) Show that $f(x)$ has a turning point at $x = 0$. Is this point a maximum, minimum or inflection point?

3. (a) Find $\int \ln x \, dx$.

(b) Find the general solution of the first order differential equation

$$\frac{dy}{dx} = 2y \ln x$$

(c) Find the solution of the first order initial value problem

$$\frac{dy}{dx} = 2y \ln x, \quad y(1) = 0.$$

4. (a) Find $\frac{d}{dx} \ln x$.
(b) Find $\frac{d}{dx} \arcsin x$.
(c) Find $\frac{d}{dx} \arctan x$.
(d) You are told that for an unknown function, f ,

$$f'(x) = f(x) + 3.$$

- (i) Show that the derivative of $f^{-1}(x)$ is $\frac{1}{x+3}$.
(ii) Find $f^{-1}(x)$.
(iii) Find $f(x)$.

5. (a) (i) Find the solution of the initial value problem

$$\cos x \frac{dy}{dx} = \frac{\sec x}{y}, \quad y\left(\frac{\pi}{2}\right) = 4.$$

- (b) You are given the following non-linear initial value problem:

$$\frac{dy}{dx} = 2^x - y^2, \quad y(0) = 1$$

- (i) Use Euler's method with steps of size 1 to estimate $y(4)$.
(ii) Use Euler's method with steps of size 2 to estimate $y(4)$.
(iii) Which of this is likely to be the best approximation?

6. (a) Find the general solution of

$$2\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 8y = 0.$$

- (b) Find the general solution of

$$2\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 8y = 4x.$$

- (c) Find the solution of

$$2\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 8y = 4x, \quad y(0) = 1, \quad y(1) = e^{-2}.$$

7. (a) You are given the following data:

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	2.0	1.8	1.2	1.1	1.8

- (b) (i) Use this data to estimate $\int_0^2 f(x) dx$ using the trapezium rule with 4 intervals.
(ii) How could the accuracy of this estimate be improved?
- (c) Find $\int \sin^5 x \cos x dx$.
- (d) (i) Find $\int \frac{3}{x^2 + x - 2} dx$.
(ii) Find the area under the curve $y = \frac{3}{x^2 + x - 2}$ between $x = 4$ and $x = 2$.
Give your answer in the form $\ln a$ where a is a real number.
- (e) For all positive numbers b , $\int_0^b g(x) dx < 0$. What does this tell you about the function $g(x)$?