MATH6103 Differential & Integral Calculus

Real Life Problems Practice Sheet Solutions

1) A projectile is fired from a cannon on flat ground at 25m/s at an angle of (approx.) 0.64^{c}. t seconds after it is fired, the x and y co-ordinates (in m) of its position are given by:

$$x = 20t$$
$$y = 15t - 5t^2$$

a) Find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.

$$\frac{dx}{dt} = 20$$
$$\frac{dy}{dt} = 15 - 10t$$

b) Using (a), find $\dfrac{dy}{dx}$ in terms of t.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dt}\frac{dt}{dx} \\ &= (15 - 10t) \cdot \frac{1}{20} \\ &= \frac{15 - 10t}{20} \\ &= \frac{3 - 2t}{4}\end{aligned}$$

c) Find the value(s) of t which make(s) $\dfrac{dy}{dx} = 0$.

$$\begin{aligned}\frac{3 - 2t}{4} &= 0 \\ 3 - 2t &= 0 \\ 3 &= 2t \\ t &= \frac{3}{2}\end{aligned}$$

d) Find the maximum height which the projectile reaches.

The maximum height will be at $t = \frac{3}{2}$:

$$\begin{aligned} y\left(\frac{3}{2}\right) &= 15 \cdot \frac{3}{2} - 5 \cdot \left(\frac{3}{2}\right)^2 \\ &= \frac{45}{2} - 5 \cdot \frac{9}{4} \\ &= \frac{45}{2} - \frac{45}{4} \\ &= \frac{45}{4} \end{aligned}$$

e) Find the values of t for which $y = 0$.

$$\begin{aligned} 15t - 5t^2 &= 0 \\ t(15 - 5t) &= 015 - 5t \qquad\qquad = 0 \text{ or } t = 0 \\ 15 &= 5t \\ t &= 3 \end{aligned}$$

So t is 3 or 0

f) How far away from the cannon does the projectile land (if it doesn't bounce)?

The projectile will land when $t = 3$:

$$\begin{aligned} x(3) &= 20 \cdot 3 \\ &= 60\text{m} \end{aligned}$$

2) The same cannon is fired but this time into a wind. This time the co-ordinates at time t are given by:

$$\begin{aligned} x &= 20t - \frac{1}{2}t^2 \\ y &= 15t - 5t^2 \end{aligned}$$

a) Find the maximum height which the projectile reaches.

The maximum height will be reached when $\frac{dy}{dt} = 0$:

$$\begin{aligned}\frac{dy}{dt} &= 15 - 10t \\ &= 0 \\ 10t &= 15 \\ t &= 1.5\end{aligned}$$

At this time the height will be:

$$\begin{aligned}y(1.5) &= 15 \cdot 1.5 - 5 \cdot 1.5^2 \\ &= 22.5 - 5 \cdot 2.25 \\ &= 22.5 - 11.25 \\ &= 11.25\end{aligned}$$

b) How far away from the cannon does the projectile land (if it doesn't bounce)?

The cannonball lands when $y = 0$:

$$\begin{aligned}15t - 5t^2 &= 0 \\ t(15 - 5t) &= 0 \\ t &= 0 \text{ or } 3\end{aligned}$$

The projectile will land at:

$$\begin{aligned}x(3) &= 20 \cdot 3 - \frac{1}{2} \cdot 3^2 \\ &= 60 - \frac{9}{2} \\ &= 55.5\end{aligned}$$