MATH6103 Differential & Integral Calculus

MATH6500 Elementary Mathematics for Engineers

Problem Sheet 9

Deadline: **Monday 14 December, 5:00**.
Hand in to **drop box 5** in the undergraduate common room (maths department, room 502).

Hand in the questions marked with an asterisk (*).

One mark will be deducted if you do not **staple your work**.

1) Solve the following:

a) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

b) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

* c) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin x$

d) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2$

e) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \sin x - \cos x$

f) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$

* g) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$

h) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x} + \sin x$

i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x - 2$

j) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$

k) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = \sin x$

l) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 0$

m) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x + 3$

n) $x\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} = x^2$

o) $4\frac{d^2y}{dx^2} - y = 8x$

* p) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$

* q) $\frac{d^2y}{dx^2} + y = x$

r) $\frac{d^2y}{dx^2} + y = \sin x$

Challenge: Adapt the method we have learned to solve this ODE:

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{3x}$$

Please turn over for questions 2 and 3.

2) [*Calculator allowed*] Use Euler's method with $h = \frac{1}{2}$ to estimate $y(1)$ when

$$\frac{dy}{dx} = \sin y + x^2$$

and

$$y(0) = 0.$$

* 3) [*Calculator allowed*] Use Euler's method with $h = \frac{1}{5}$ to estimate $y(1)$ when

$$\frac{dy}{dx} = \sin y + x^2$$

and

$$y(0) = 0.$$