MATH6103 Differential & Integral Calculus

MATH6500 Elementary Mathematics for Engineers

Problem Sheet 8

Deadline: **Monday 7 December, 5:00**.
Hand in to **drop box 5** in the undergraduate common room (maths department, room 502).

Hand in the questions marked with an asterisk (*).

One mark will be deducted if you do not **staple your work**.

1) Use seperation of variables to solve the following:

a) $\frac{dy}{dx} + \frac{k}{x^2} = 0$

b) $\frac{dy}{dx} = \frac{y}{x}$

* c) $\frac{dy}{dx} = \frac{x}{y}$

* d) $\frac{dx}{dy} + \frac{k}{x^2} = 0$

* e) $\sin y \cos x \frac{dy}{dx} = \sin x \cos y$

f) $\frac{1+y^2}{(1+x^2)xy} = \frac{dy}{dx}$

g) $xy\frac{dy}{dx} = x$

h) $x\sqrt{y^2-1} - y\sqrt{x^2-1}\frac{dy}{dx} = 0$

2) Use integrating factors to solve the following:

* a) $\frac{dy}{dx} - 2xy = 2x$

b) $\frac{dy}{dx} = y - x$

c) $\frac{dy}{dx} + y\tan x = 1$

* d) $\frac{dy}{dx} + ay = e^x$ (a is a constant)

e) $x\frac{dy}{dx} - ay = x + 1$

* f) $1 - ye^x = e^x\frac{dy}{dx}$

g) $t^2\frac{dx}{dt} + xt + 1 = 0$

h) $\cos^2 x\frac{dy}{dx} + y = \tan x$

Please turn over for question 3.

3) Newton's Law of Cooling states that

$$\frac{dT}{dt} + kT = k\theta,$$

where T is the temperature (in °C) of a body at time t (seconds), θ is the ambient temperature (a constant, measured in °C) and k is a constant.

In an experiment, a body starts at 10°C. After 10 seconds, it has reached 20°C. After 20 seconds, it has reached 25°C.

* 3a Show that $e^{-10k} = \dfrac{\theta - 20}{\theta - 10}$ and $e^{-20k} = \dfrac{\theta - 25}{\theta - 10}$.

* 3b Show that $\left(\dfrac{\theta - 20}{\theta - 10}\right)^2 = \dfrac{\theta - 25}{\theta - 10}$.

* 3c Find the ambient temperature.