MATH6103 Differential & Integral Calculus

MATH6500 Elementary Mathematics for Engineers

Problem Sheet 5 solutions

1) Find:

a) $\int 4x + 5x^4 \; dx$

$$2x^2 + x^5 + c$$

b) $\int 3x^3 - 8x^2 \; dx$

$$\frac{3x^4}{4} - \frac{8x^3}{3} + c$$

c) $\int e^x - \sin x \; dx$

$$e^x + \cos x + c$$

d) $\int \sin(3x+4) \; dx$

Let $u = 3x + 4$.

$$\begin{aligned} \frac{du}{dx} &= 3 \\ \frac{1}{3}\; du &= \; dx \\ \int \sin(3x+4)\; dx &= \frac{1}{3}\int \sin u \; du \qquad = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos(3x+4) + c \end{aligned}$$

e) $\int \cos(8x-4) \; dx$

Let $u = 8x - 4$.

$$\begin{aligned} \frac{du}{dx} &= 8 \\ \frac{1}{8}\; du &= \; dx \\ \int \cos(8x-4)\; dx &= \frac{1}{8}\int \cos u \; du \\ &= \frac{1}{8}\sin u + c \\ &= \frac{1}{8}\sin(8x-4) + c \end{aligned}$$

f) $\int \tan x \; dx$

$$
\begin{aligned}
\int \tan x \ dx &= \int \frac{\sin x}{\cos x} \ dx \\
&= - \int \frac{\frac{d}{dx}(\cos x)}{\cos x} \ dx \\
&= - \ln|\cos x| + c
\end{aligned}
$$

g) $\int \sin x + \sec^2 x \ dx$

$$-\cos x + \tan x + c$$

h) $\int (7x+7)^2 \ dx$

$$\frac{(7x+7)^3}{21} + c$$

i) $\int_0^2 (7x+7)^2 \ dx$

$$
\begin{aligned}
\left[\frac{(7x+7)^3}{21}\right]_0^2 &= \left(\frac{(7\cdot 2+7)^3}{21}\right) - \left(\frac{(7\cdot 0+7)^3}{21}\right) \\
&= \left(\frac{21^3}{21}\right) - 0 \\
&= 21^2 \\
&= 441
\end{aligned}
$$

j) $\int \frac{1}{2x+1} \ dx$

$$\frac{1}{2}\int \frac{2}{2x+1} \ dx$$

$$\frac{1}{2}\int \frac{\frac{d}{dx}(2x+1)}{2x+1} \ dx$$

$$\frac{1}{2}\ln|2x+1| + c$$

k) $\int \sqrt{4x-8} \ dx$

Let $u = 4x - 8$.

$$\frac{1}{4} \ du = \ dx$$

$$
\begin{aligned}
\int \sqrt{4x-8}\ dx &= \frac{1}{4}\int \sqrt{u}\ du \\
&= \frac{1}{4}\int u^{\frac{1}{2}}\ du \\
&= \frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} + c \\
&= \frac{u^{\frac{3}{2}}}{6} + c
\end{aligned}
$$

l) $\int_0^{\pi/2} \cos x\sqrt{\sin x}\ dx$

Let $u = \sin x$.

$$du = \cos x\ dx$$

$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} \cos x\sqrt{\sin x}\ dx &= \int_{x=0}^{x=\frac{\pi}{2}} \sqrt{u}\ du \\
&= \int_{x=0}^{x=\frac{\pi}{2}} u^{\frac{1}{2}}\ du \\
&= \left[\frac{2}{3}u^{\frac{3}{2}}\right]_{x=0}^{x=\frac{\pi}{2}} \\
&= \left[\frac{2}{3}(\sin x)^{\frac{3}{2}}\right]_0^{\frac{\pi}{2}} \\
&= \left(\frac{2}{3}\left(\sin\frac{\pi}{2}\right)^{\frac{3}{2}}\right) - \left(\frac{2}{3}(\sin 0)^{\frac{3}{2}}\right) \\
&= \left(\frac{2}{3}(1)^{\frac{3}{2}}\right) - 0 \\
&= \frac{2}{3}
\end{aligned}
$$

2) A projectile is fired vertically upwards.

Its velocity, $v(t)$, (in ms^{-1}) at time t (in s) is given by:

$$v(t) = 50 - 9.8t$$

Calculate:

a) the time, T_0, at which the projectile's velocity is 0.

$v(t) = 0$ when:

$$\begin{aligned}
50 - 9.8t &= 0 \\
9.8t &= 50 \\
t &= \frac{50}{9.8} \\
t &= \frac{250}{49}
\end{aligned}$$

b) the distance the projectile travels in the first 5 seconds.

$$\begin{aligned}
\int_0^5 v(t)\ dt &= \int_0^5 50 - 9.8t\ dt \\
&= \left[50t - 4.9t^2\right]_0^5 \\
&= \left(50 \cdot 5 - 4.9 \cdot 5^2\right) - \left(50 \cdot 0 - 4.9 \cdot 0^2\right) \\
&= (250 - 4.9 \cdot 25) - 0 \\
&= 252.5
\end{aligned}$$

c) the highest height which the projectile reaches.

$$\begin{aligned}
\int_0^{\frac{250}{49}} v(t)\ dt &= \int_0^{\frac{250}{49}} 50 - 9.8t\ dt \\
&= \left[50t - 4.9t^2\right]_0^{\frac{250}{49}} \\
&= \left(50 \cdot \frac{250}{49} - 4.9 \cdot \left(\frac{250}{49}\right)^2\right) - \left(50 \cdot 0 - 4.9 \cdot 0^2\right) \\
&= \frac{50 \cdot 250}{49} - \frac{250 \cdot 250 \cdot 4.9}{49 \cdot 49} \\
&= \frac{50 \cdot 250}{49} - \frac{250 \cdot 250}{49 \cdot 10} \\
&= \frac{50 \cdot 250}{49} - \frac{250 \cdot 25}{49} \\
&= \frac{25 \cdot 250}{49} \\
&= \frac{6250}{49}
\end{aligned}$$