

MATH6103 Differential & Integral Calculus  
MATH6500 Elementary Mathematics for Engineers

## Problem Sheet 3

Deadline: **Monday 26 October, 5:00.**

Hand in to **drop box 5** in the undergraduate common room (maths department, room 502).

**Hand in the questions marked with an asterisk (\*).**

One mark will be deducted if you do not **staple your work**.

1) Differentiate the following functions:

a)  $f(x) = x^2 - x^3$

f)  $k(x) = \sqrt{x}$

b)  $g(x) = 4x^{\frac{1}{2}}$

g)  $l(x) = \cos^2 x$  [Hint: chain rule]

c)  $h(x) = \cos x$

h)  $m(x) = \cos x \sin x$  [Hint: product rule]

d)  $i(x) = \frac{1}{x}$

i)  $n(x) = x \sin x$  [Hint: product rule]

e)  $j(x) = 359x^{17}$

\* j)  $o(x) = \sqrt{3x^2 + 8}$  [Hint: chain rule]

2) Differentiate the following functions:

a)  $p(x) = \cos(x + 3)$

e)  $u(x) = \sin^2 x$

b)  $q(x) = x^3 \sin x$

f)  $v(x) = \sin^2 x \cos x$

c)  $r(x) = \sin(x^2 - 1)$

\* g)  $w(x) = \sqrt{\sin x + \cos x}$

\* d)  $s(x) = x \sin x \cos x$

h)  $y(x) = (x^{10} - x^2 \sin x)^2$

3) Find the  $x$  co-ordinates of the points where  $f'(x) = 0$ :

\* a)  $f(x) = \sin x$

b)  $f(x) = 2x^3 - 15x^2 + 36x - 8$

\* c)  $f(x) = x^3 - 9x$

d)  $f(x) = \sin x + x$

e)  $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - 8x^2 + 16x + 42$

**Challenge:** Use the product and chain rules to show that:

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

[Hint: Use  $\frac{f(x)}{g(x)} = f(x)(g(x))^{-1}$ ]